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LETTERS TO THE EDITOR

ON THE DYNAMIC STABILITY OF VISCOELASTIC COLUMNS

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1. INTRODUCTION

We would like to refer here to a previous paper published by us [1], in which the dynamic stability of a viscoelastic column subjected to a periodic longitudinal load was investigated. The viscoelastic behavior was given in terms of the Boltzmann superposition principle, yielding an integro-differential equation of motion. The stability boundaries of this equation were determined *analytically* by using the multiple-scales method. It was shown there that the stability properties of the viscoelastic column are time dependent. This implies, for example, that an initially stable system can turn unstable after a finite time, unlike columns that are described by the elastic model with viscous damping. However, in a recent *numerical* investigation we could not re-confirm these results. In the following we would like to present our new outcomes concerning this problem.

2. STABILITY PROPERTIES OF A VISCOELASTIC COLUMN

The integro-differential equation of motion derived for the problem under consideration is (see equation (9) in reference [1])

$$\ddot{f}(t) + \omega^2[1 - 2\eta\cos(\theta t)]f(t) = -\omega^2\int_0^t \dot{D}(t-t')f(t')\mathrm{d}t'. \tag{1}$$

The integral on the right-hand side can be performed in parts to yield:

$$\int_0^t \dot{D}(t-t')f(t')\mathrm{d}t' = \dot{D}(0)\int_0^t f(t')\mathrm{d}t' - \int_0^t \ddot{D}(t-t')\int_0^{t'} f(t'')\mathrm{d}t''\mathrm{d}t'. \tag{2}$$

The first term on the right-hand side of equation (2) is of order δ while the second one is of δ^2. In addition, the second term is also bounded as $t \to \infty$ and hence may be neglected within the analysis performed in reference [1]. Hence, equation (11) therein should be

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$$\ddot{f}(t) + \omega^2[1 - 2\eta\cos(\theta t)]f(t) = -\delta\omega^2\dot{D}(0)\int_0^t f(t')\mathrm{d}t' \tag{3}$$

and by repeating the analysis presented in [1] we find that the stability boundaries are given by

$$\omega^2 = \frac{\theta^2}{4}\left[1 \pm \sqrt{\eta^2 - \frac{4}{\theta^2}[\dot{D}(0)]^2}\right]. \tag{4}$$

The critical (minimum) value of the excitation parameter, η_c, for which instability occurs is

$$\eta_c = \frac{2}{\theta}|\dot{D}(0)|. \tag{5}$$

To conclude, it is seen that all the stability properties of the problem are *not* time-dependent. In addition, if the system is stable at zero time it will remain so indefinitely. These properties were confirmed in the numerical investigation presented in reference [2].

Finally, consider the example presented in reference [1] of a column made of a viscoelastic material modelled as the Standard Linear Solid, for which

$$D(t) = a + b\mathrm{e}^{-\beta t}, \tag{6}$$

where a, b and β are constants. For this case

$$\eta_c = \frac{2}{\theta}\beta b \tag{7}$$

and the stability boundaries are given by

$$\omega^2 = \frac{\theta^2}{4}\left[1 \pm \sqrt{\eta^2 - \eta_c^2}\right] \quad \text{for} \quad \eta \geq \eta_c. \tag{8}$$

REFERENCES

1. G. Cederbaum and M. Mond 1992 *Journal of Applied Mechanics* **59**, 16–19. Stability properties of a viscoelastic column under a periodic force.
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